MATHEMATICS

Textbook for Class XI

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विद्यया ऽमृतमश्नुते

एन सी ई आर टी
NCERT

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

Foreword

The National Curriculum Framework (NCF), 2005, recommends that children's life at school must be linked to their life outside the school. This principle marks a departure from the legacy of bookish learning which continues to shape our system and causes a gap between the school, home and community. The syllabi and textbooks developed on the basis of NCF signify an attempt to implement this basic idea. They also attempt to discourage rote learning and the maintenance of sharp boundaries between different subject areas. We hope these measures will take us significantly further in the direction of a child-centred system of education outlined in the National Policy on Education (1986).

The success of this effort depends on the steps that school principals and teachers will take to encourage children to reflect on their own learning and to pursue imaginative activities and questions. We must recognise that given space, time and freedom, children generate new knowledge by engaging with the information passed on to them by adults. Treating the prescribed textbook as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. Inculcating creativity and initiative is possible if we perceive and treat children as participants in learning, not as receivers of a fixed body of knowledge.

These aims imply considerable change in school routines and mode of functioning. Flexibility in the daily time-table is as necessary as rigour in implementing the annual calendar so that the required number of teaching days are actually devoted to teaching. The methods used for teaching and evaluation will also determine how effective this textbook proves for making children's life at school a happy experience, rather than a source of stress or boredom. Syllabus designers have tried to address the problem of curricular burden by restructuring and reorienting knowledge at different stages with greater consideration for child psychology and the time available for teaching. The textbook attempts to enhance this endeavour by giving higher priority and space to opportunities for contemplation and wondering, discussion in small groups, and activities requiring hands-on experience.

The National Council of Educational Research and Training (NCERT) appreciates the hard work done by the Textbook Development Committee responsible for this

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Director
National Council of Educational
Research and Training

New Delhi
20 December 2005

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Contents

Foreword *iii*

1. Sets **1**

1.1 Introduction 1
1.2 Sets and their Representations 1
1.3 The Empty Set 5
1.4 Finite and Infinite Sets 6
1.5 Equal Sets 7
1.6 Subsets 9
1.7 Power Set 12
1.8 Universal Set 12
1.9 Venn Diagrams 13
1.10 Operations on Sets 14
1.11 Complement of a Set 18
1.12 Practical Problems on Union and Intersection of Two Sets 21

2. Relations and Functions **30**

2.1 Introduction 30
2.2 Cartesian Product of Sets 30
2.3 Relations 34
2.4 Functions 36

3. Trigonometric Functions **49**

3.1 Introduction 49
3.2 Angles 49
3.3 Trigonometric Functions 55
3.4 Trigonometric Functions of Sum and Difference of Two Angles 63
3.5 Trigonometric Equations 74

4. Principle of Mathematical Induction **86**

4.1 Introduction 86
4.2 Motivation 87
4.3 The Principle of Mathematical Induction 88

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5. Complex Numbers and Quadratic Equations **97**
5.1 Introduction 97
5.2 Complex Numbers 97
5.3 Algebra of Complex Numbers 98
5.4 The Modulus and the Conjugate of a Complex Number 102
5.5 Argand Plane and Polar Representation 104
5.6 Quadratic Equations 108

6. Linear Inequalities **116**
6.1 Introduction 116
6.2 Inequalities 116
6.3 Algebraic Solutions of Linear Inequalities in One Variable and their Graphical Representation 118
6.4 Graphical Solution of Linear Inequalities in Two Variables 123
6.5 Solution of System of Linear Inequalities in Two Variables 127

7. Permutations and Combinations **134**
7.1 Introduction 134
7.2 Fundamental Principle of Counting 134
7.3 Permutations 138
7.4 Combinations 148

8. Binomial Theorem **160**
8.1 Introduction 160
8.2 Binomial Theorem for Positive Integral Indices 160
8.3 General and Middle Terms 167

9. Sequences and Series **177**
9.1 Introduction 177
9.2 Sequences 177
9.3 Series 179
9.4 Arithmetic Progression (A.P.) 181
9.5 Geometric Progression (G.P.) 186
9.6 Relationship Between A.M. and G.M. 191
9.7 Sum to *n* terms of Special Series 194

10. Straight Lines **203**
10.1 Introduction 203
10.2 Slope of a Line 204
10.3 Various Forms of the Equation of a Line 212
10.4 General Equation of a Line 220
10.5 Distance of a Point From a Line 225

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11. Conic Sections **236**
11.1 Introduction 236
11.2 Sections of a Cone 236
11.3 Circle 239
11.4 Parabola 242
11.5 Ellipse 247
11.6 Hyperbola 255

12. Introduction to Three Dimensional Geometry **268**
12.1 Introduction 268
12.2 Coordinate Axes and Coordinate Planes in Three Dimensional Space 269
12.3 Coordinates of a Point in Space 269
12.4 Distance between Two Points 271
12.5 Section Formula 273

13. Limits and Derivatives **281**
13.1 Introduction 281
13.2 Intuitive Idea of Derivatives 281
13.3 Limits 284
13.4 Limits of Trigonometric Functions 298
13.5 Derivatives 303

14. Mathematical Reasoning **321**
14.1 Introduction 321
14.2 Statements 321
14.3 New Statements from Old 324
14.4 Special Words/Phrases 329
14.5 Implications 335
14.6 Validating Statements 339

15. Statistics **347**
15.1 Introduction 347
15.2 Measures of Dispersion 349
15.3 Range 349
15.4 Mean Deviation 349
15.5 Variance and Standard Deviation 361
15.6 Analysis of Frequency Distributions 372

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16. Probability **383**
16.1 Introduction 383
16.2 Random Experiments 384
16.3 Event 387
16.4 Axiomatic Approach to Probability 394

Appendix 1: Infinite Series **412**
A.1.1 Introduction 412
A.1.2 Binomial Theorem for any Index 412
A.1.3 Infinite Geometric Series 414
A.1.4 Exponential Series 416
A.1.5 Logarithmic Series 419

Appendix 2: Mathematical Modelling **421**
A.2.1 Introduction 421
A.2.2 Preliminaries 421
A.2.3 What is Mathematical Modelling 425

Answers **433**

Supplementary Material **466**

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BE A STUDENT OF STUDENTS

A teacher who establishes rapport with the taught, becomes one with them, learns more from them than he teaches them. He who learns nothing from his disciples is, in my opinion, worthless. Whenever I talk with someone I learn from him. I take from him more than I give him. In this way, a true teacher regards himself as a student of his students. If you will teach your pupils with this attitude, you will benefit much from them.

Talk to Khadi Vidyalaya Students, Sevagram
Harijan Seva, 15 February 1942 (CW 75, p. 269)

USE ALL RESOURCES TO BE CONSTRUCTIVE AND CREATIVE

What we need is educationists with originality, fired with true zeal, who will think out from day to day what they are going to teach their pupils. The teacher cannot get this knowledge through musty volumes. He has to use his own faculties of observation and thinking and impart his knowledge to the children through his lips, with the help of a craft. This means a revolution in the method of teaching, a revolution in the teachers' outlook. Up till now you have been guided by inspector's reports. You wanted to do what the inspector might like, so that you might get more money yet for your institutions or higher salaries for yourselves. But the new teacher will not care for all that. He will say, '1 have done my duty to my pupil if I have made him a better man and in doing so I have used all my resources. That is enough for me'.

Harijan, 18 February 1939 (CW 68, pp. 374-75)

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