

# MATHEMATICS

*Textbook for Class XI*



राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्  
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

## Foreword

The National Curriculum Framework (NCF), 2005, recommends that children's life at school must be linked to their life outside the school. This principle marks a departure from the legacy of bookish learning which continues to shape our system and causes a gap between the school, home and community. The syllabi and textbooks developed on the basis of NCF signify an attempt to implement this basic idea. They also attempt to discourage rote learning and the maintenance of sharp boundaries between different subject areas. We hope these measures will take us significantly further in the direction of a child-centred system of education outlined in the National Policy on Education (1986).

The success of this effort depends on the steps that school principals and teachers will take to encourage children to reflect on their own learning and to pursue imaginative activities and questions. We must recognise that given space, time and freedom, children generate new knowledge by engaging with the information passed on to them by adults. Treating the prescribed textbook as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. Inculcating creativity and initiative is possible if we perceive and treat children as participants in learning, not as receivers of a fixed body of knowledge.

These aims imply considerable change in school routines and mode of functioning. Flexibility in the daily time-table is as necessary as rigour in implementing the annual calendar so that the required number of teaching days are actually devoted to teaching. The methods used for teaching and evaluation will also determine how effective this textbook proves for making children's life at school a happy experience, rather than a source of stress or boredom. Syllabus designers have tried to address the problem of curricular burden by restructuring and reorienting knowledge at different stages with greater consideration for child psychology and the time available for teaching. The textbook attempts to enhance this endeavour by giving higher priority and space to opportunities for contemplation and wondering, discussion in small groups, and activities requiring hands-on experience.

The National Council of Educational Research and Training (NCERT) appreciates the hard work done by the Textbook Development Committee responsible for this

book. We wish to thank the Chairperson of the advisory group in Science and Mathematics, Professor J.V. Narlikar and the Chief Advisor for this book Professor P.K. Jain for guiding the work of this committee. Several teachers contributed to the development of this textbook; we are grateful to their principals for making this possible. We are indebted to the institutions and organisations which have generously permitted us to draw upon their resources, material and personnel. We are especially grateful to the members of the National Monitoring Committee, appointed by the Department of Secondary and Higher Education, Ministry of Human Resource Development under the Chairpersonship of Professor Mrinal Miri and Professor G.P. Deshpande, for their valuable time and contribution. As an organisation committed to the systemic reform and continuous improvement in the quality of its products, NCERT welcomes comments and suggestions which will enable us to undertake further revision and refinement.

New Delhi  
20 December 2005

*Director*  
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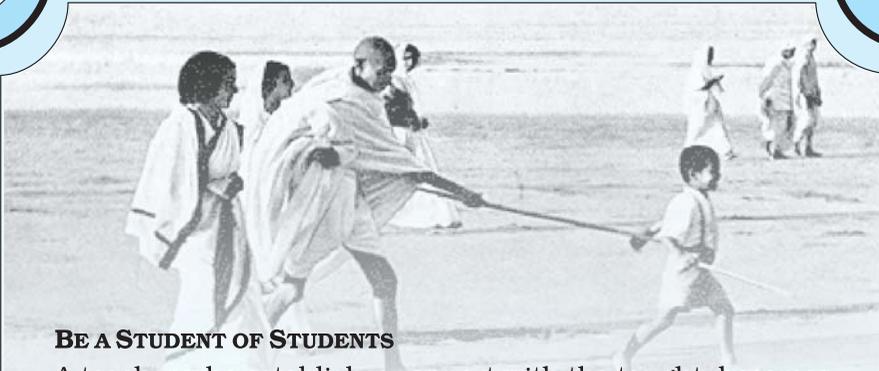
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#### **BE A STUDENT OF STUDENTS**

A teacher who establishes rapport with the taught, becomes one with them, learns more from them than he teaches them. He who learns nothing from his disciples is, in my opinion, worthless. Whenever I talk with someone I learn from him. I take from him more than I give him. In this way, a true teacher regards himself as a student of his students. If you will teach your pupils with this attitude, you will benefit much from them.

Talk to Khadi Vidyalaya Students, Sevagram  
*Harijan Seva, 15 February 1942 (CW 75, p. 269)*

#### **USE ALL RESOURCES TO BE CONSTRUCTIVE AND CREATIVE**

What we need is educationists with originality, fired with true zeal, who will think out from day to day what they are going to teach their pupils. The teacher cannot get this knowledge through musty volumes. He has to use his own faculties of observation and thinking and impart his knowledge to the children through his lips, with the help of a craft. This means a revolution in the method of teaching, a revolution in the teachers' outlook. Up till now you have been guided by inspector's reports. You wanted to do what the inspector might like, so that you might get more money yet for your institutions or higher salaries for yourselves. But the new teacher will not care for all that. He will say, 'I have done my duty to my pupil if I have made him a better man and in doing so I have used all my resources. That is enough for me'.

*Harijan, 18 February 1939 (CW 68, pp. 374-75)*

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